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Demand, Supply and Coordination:

An Integrated Theory of the Division of Labor

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Abstract

Product demand, supply and internal coordination are all explicitly specified in a model to study how they jointly determine the division of labor (job span). A larger job span means fewer workers are used to cover a production process, which is helpful in coordination and product quality, but not in lowering training cost. Although coordination is at the core, the model shows that, in general, job span is affected by all demand and supply factors. With marginal labor productivity declining, job span is narrower when the market is larger, as Adam Smith believed. It is narrower when coordination technology is better or wage is lower. It is likely narrower when unit training cost or productivity is higher. The results are reversed if labor has increasing marginal productivity. These results are either new or shed new light on previous theories of specialization. They have plausible empirical implications. They show the importance of an integrated approach to the study of job design.

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1. Introduction

In this paper, we study a very old question in economics with a new theoretic model. The multi-century old question is what determines the division of labor. The model we use to study the question is new in that it explicitly specifies product demand, supply and internal coordination to see how they simultaneously determine the division of labor, rather than looking at one of these factors at a time as previous studies did.

The model contributes significantly to the literature in at least three ways. First, it fills a gap in the literature due to the lag of theories behind empirical findings. Second, it sheds new light on some of the best-known previous claims on the division of labor, e.g., those made by Adam Smith (1776) and Becker and Murphy (1992). We can thus either further qualify or generalize these claims, or both. Last, but not least, the model has new, interesting and plausible empirical implications that previous theories have not had.

Since the early 1990s, a large and rich body of empirical literature of human resource practices including job design has emerged and provided convincing evidence of a close relationship between the external market and job design. Baily (1993), Berg et al (1994), and Dunlop and Weil (1996) reported that, in the US apparel industry, adoption of group-based job design with more tasks for individual workers in modular assembly systems is more likely in the production of fashion wears where delivery time and quality pressures are high, while the traditional, individual-based and narrow job design remains dominant in the production of apparels for mass consumption. A similar experience is reported by Geary (1999) in apparel manufacturing in Scotland. Geber (1992) reported that teamwork at Saturn was intended to solve General Motor's problem of poor product quality and declining market share due to it.[1] Osterman (1994) finds systematic evidence "strongly confirming" that a market with emphases on "service, quality, and variety of products rather than low cost" are among the most important variables determining who would adopt innovative human resource practices which typically have broadly defined jobs as a core element. Along with many others, these findings make evident two things. On the one hand, the external market plays a critical role in determining job design. At the same time, job design affects the demand for a product because it has implications for product quality, variety and delivery time.[2]

The close and interactive relationship between the external market and job design has not been carefully studied by theorists as they have focused on one factor at a time. Recognizing the great importance of the division of labor to productivity, Adam Smith (1776) asked the question of what determines the division of labor. He conjectures it is the size of the market. Becker and Murphy (1992) disagree and argue that the division of labor is mainly limited by the cost of coordination. They show that, only in the trivial case where the marginal benefit is always greater than the marginal cost of specialization does the size of the market matter. Also focusing on coordination, Bolton and Dewatripont (1994) modeled the efficiency gain resulting from specialized information processing and the increased cost of communication with more specialization. A most recent and significant contribution along this line is made by Dessein and Santos (2003),

[1] Also see Lazear (1998, p.515) on Saturn's experience.
[2] Sociologists who study organizations are keen to the interdependence between an organization and its environment. See Bedeian and Zammuto (1991, Ch. 8) for a presentation of different theories on this. Our integrated model has elements of contingency theory and strategic choice theory from sociology, with the firm responding to its external market and also trying to influence the market through job design.

who derived the demand for coordination when workers can be coordinated either by *ex ante* rules or through *ex post* communication.

The gist of our integrated model can be summarized as follows. As in Bolton and Dewatripont (1994) and others, the model assumes that coordination is more difficult when jobs are more specialized, resulting in lower expected product quality and thereby reduced demand for the product. The desire to improve product quality motivates the firm to increase job span.[3] However, to be able to perform a job of broader span, a worker needs more training. Saving on training cost motivates the firm to reduce job span. Equilibrium job span is found when the marginal benefit of it, seen in better product quality, is equal to the marginal cost of it, seen in higher training cost. When a demand or supply condition changes, the relative significance of one these margins to the other is affected, leading to a new equilibrium job span.

The model finds that, with marginal productivity of labor declining, jobs do become more specialized when the market becomes larger, as Smith (1776) believed. The reason for this is that, when production is at a larger scale and involves more workers, a marginal increase in per worker training results in a larger marginal increase in total training cost, meaning that the firm can reduce job span for a more significant saving of total training cost. In light of this logic, we can make a more general statement: Any change in demand or supply leading to a larger equilibrium output is a reason for increased division of labor. Adam Smith's (1776) claim of the division of labor depending on market size is a special case of this general statement. However, the result

[3] We discuss later in the paper the effect of product delivery time or variety rather than quality on job design. By job design, we mean how broad or narrow the job span is in a firm. A narrower job span means a greater degree of the division of labor with fewer tasks bundled into the job and assigned for a worker to perform. We use these terms interchangeably in this paper.

is reversed when marginal productivity of labor is increasing. Market size does not affect job span when marginal productivity of labor is constant. Constant marginal productivity of labor is exactly what Becker and Murphy (1992) assumed, implicitly, in their model. It is thus not surprising that they find specialization generally having nothing to do with the size of the market. We will explain these results after they are formally derived.

On the supply side, the model finds that a higher (market-determined) wage leads to a larger job span. The reason is that higher labor cost leads to a lower output level. This changes the cost and benefit of specialization in favor of a larger job span as explained in the preceding paragraph. This result has the empirical implication that expensive labor should not be hired to produce low quality product, which seems quite plausible and offers another explanation for why quality is usually better when a product is produced in a developed rather than a developing country. The model also finds that higher unit training cost reduces job span so long as unit training cost is small relative to the size of the market. Similarly, lower labor productivity is likely to lead to a larger job span. Better coordination technology always leads to a smaller job span, as in Becker and Murphy (1992) and Bolton and Dewatripont (1994).[4]

Both conceptually and technically, this paper is most closely related to the literature studying coordination problems in organizations, especially those in the literature studying horizontal specialization. As further explained later in the paper, this paper also contributes to human capital theory and the comparative study of organizations. It abstracts from incentive problems in organizations.

[4] This result is not in Dessein and Santos (2003). In their model, improved coordination technology may lead the firm to increase worker discretion at work. Consequently, demand for *ex post* coordination may increase, leading to a larger job span.

The paper is organized as follows. The model is introduced in section 2. The equilibrium job span is derived in section 3. Comparative statics of job span with respect to all the factors of demand, supply and internal coordination are studied in sections 4 through 6. Section 7 discusses the model. Section 8 reviews the literature and this paper's relation to it. Section 9 ends the paper with some additional comments.

2. The Model:

Our model has the following specifications

Production and Job Span

A production line (process) has unit length in the interval of [0, 1] and uses labor as the only input. Let N be the technologically determined maximum number of workers that the firm can assign to a line, N being a very large positive number. Beyond N, the line becomes too crowded and experiences a dramatic loss of productivity. The firm assigns n workers, $0 < n \leq N$, to cover the entire line. This way, the firm divides the line into n jobs. Wage is w per worker and determined exogenously by the supply and demand in the labor market.

Assuming symmetry among all workers and all jobs, the span of a job is $b = 1/n$. Job 1 has the responsibility for the part of the process in the range of $(0, b_1]$, job 2 has the responsibility for the part of the process in the range of $(b_1, b_2]$, and so on. In general, b_i denotes job i with the responsibility for the segment of the production line in the range of $(ib-b, ib]$, $i \in [1, n]$. A larger value of b represents a larger job span, which means that more tasks are grouped in the job. We will ignore the fact that the number of tasks is usually discrete whereas b is a continuous variable, and use the terms "more tasks" and "a larger job span" loosely and interchangeably. The integer issue is discussed in section 7 of the paper.

A worker has a total amount of time t=1. When b is larger, the worker has a broader range of tasks to perform and each task receives less time. Per task output is thus reduced, as in Becker and Murphy (1992), and so is per line output. This implies a positive relationship of the output of a production line q_{line} to the number of workers assigned to the line. For simplicity, assume that this relationship is linear in the form of

$$q_{line} = n = 1/b.$$

The simplifying assumption does not lead to a loss of generality. Its essence is that per line output is proportional to the number of workers assigned to the line.

Note the feature of constant marginal productivity of labor in the production of q_{line} in the range of $n \in [0, N]$. The assumption is largely for convenience, since in our model there is a gain to specialization reflected in lower per worker training cost as we will specify shortly. Qualitatively, the results we obtain are not affected when marginal productivity changes are not too dramatically.[5]

A parallel line can be established to increase output. The firm hires L workers to set up a total of L/n parallel lines. b_i on different lines can be seen as different positions in a job of the same title. A firm with a job span of breath b_i has a total of $n = 1/b_i$ jobs and L/n positions for each job.

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[5] Non-dramatic changes in marginal productivity of labor serve to make the issue of job design nontrivial. If marginal productivity decreases or increases rapidly, this factor alone would determine job design. In reality, it is quite possible that, in many situations and for the reasons identified by Adam Smith (1776), the number of workers assigned to a production line would initially lead to significant gains. When this is true, the firm would assign more workers to the line. But, beyond a certain point (say N as we have assumed), productivity will decline rapidly. The number of workers assigned to the line should never be beyond this point. In between these

With per line output being q_{line}= n, and the number of lines L/n, it follows that total output from all the lines is $q_{all\ lines}$= n(L/n) = L.

$q_{all\ lines}$ is used as an input in combination with all other inputs (or, for simplicity, *the* other input K) to produce the final output q. Assume that the production of the final product has a Cobb-Douglas functional form with all the conventional features, we have

$$q = aK^{\alpha}(q_{all\ lines})^{\beta} \qquad (1)$$

where a > 0 is the coefficient, $0 < \beta < 1$. We assume that K is fixed and let $\theta' = aK^{\alpha}$. This assumption makes K an exogenous factor in the model. This is reasonable if K is a factor the firm cannot change, either in the short run or even in the long run. Examples of such a factor include physical capital, managerial or distribution capacity, limited supply of a certain input. Later in the paper, we will study how an exogenous change in θ', which could be due to a change in a, K or α individually or jointly, affects job design.

It is very important not to confuse constant marginal productivity of labor in the production of $q_{per\ line}$ with declining marginal productivity of labor in the production of q reflected in $\beta < 1$. The former is about how workers are assigned to jobs, when the total number of workers is given. The Latter is how output changes when total employment changes. It could be caused by more aggravated monitoring and incentive problems as the total number of workers increases, as in Qian (1994), even when job design remains optimal. Another possible cause of it is a more aggregated problem of limited managerial attention as the total number of workers increases, as in Cremer (1980) and Geanakoplos and Milgrom (1991). Again, the problem is a function of the size of the firm even when job design remains optimal.

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two extremes, there is likely a range in which productivity is relatively stable. The equilibrium number of workers assigned to a line should always be in this range.

Assume $\beta = 0.5$, and substitute L in for $q_{all\ lines}$. The production function becomes $q = \theta' L^{1/2}$, which has the inverse form of

$$L = \theta q^2, \qquad \text{where } \theta = 1/\theta'. \qquad (2)$$

Assuming $\beta = 0.5$ is largely for convenience. As we will explain later, it does not qualitatively affect the results of the model, so long as $\beta < 1$. A larger θ means less of the other input used in production, or the other factor has become less productive, making L (or, more precisely, the intermediate product $q_{all\ lines}$) less productive in the production of the final output q.

Training

Without training, a worker has zero productivity. The amount of training needed to perform a job, h, is proportional to the job span b, i.e., $h = \lambda b$, where $\lambda > 0$ is a constant.

Without any loss of generality, assume $\lambda = 1$, leading to $h = b$, i.e., the amount of training needed is equal to the span of the job that the worker is assigned to perform.

Training has unit cost c. The total cost of training a worker to perform a job is thus $ch = cb = c/n$.

Since there are n workers in a production line, total training cost to establish a complete production line of unit length is $n(c/n) = c$, which captures the cost of a larger job span. When job span is larger, more parallel lines need to be established to produce a given amount of output, leading to a larger total training cost.[6]

Coordination and product quality

This part of our model is a modified version of Dessein and Santos (2003). Like them, we assume that the local work environment of a worker is uncertain. The random

[6] Berg, et al (1994) see higher training cost as one of the main barriers to the adoption of team-oriented modular assembly system in the US apparel industry.

variable in the local environment of a job is independently distributed across jobs and its realized value η_i observed only by worker i.

In Dessein and Santos (2003), product quality is determined by two sets of worker actions. First, it is determined by how well a worker's action a_{ii} matches with the local condition η_i. $(a_{ii} - \eta_i)^2$, $i \in n$ is the measure of the effectiveness of the match. Second, quality is also determined by how workers' actions are matched with each other. Let a_{ij} be i's action to match with a_{jj}. $(a_{ij} - a_{jj})^2$, $i, j \in n$, $j \neq i$, is the measure of effectiveness of this match. The larger the value of $(a_{ij} - a_{jj})^2$, the worse is the match. Similarly, $(a_{ji} - a_{ii})^2$ is the measure of the effectiveness of j's action to match with a_{ii}. Having these two separate sets of actions by every worker is critical in their model in order to derive endogenous demand for *ex post* coordination.

To focus on the problem of cross-worker coordination, we assume that matching the action with the local condition is not a problem for a worker. *Ex post* coordination is always needed and the only challenge. We therefore ignore the first set of actions and focus only on the second set of actions. It is very important that we bear in mind that $a_{jj} = \eta_j$ is a value first known to j only and it needs to be communicated to i for $(a_{ij} - a_{jj})^2$ to be small. And this is true between any two workers. Under this assumption, aggregate ineffectiveness in the system is measured by[7]

[7] We have assumed person-to-person (or job-to-job), rather than task-to-task, communication as in Dessein and Satos (1993). In the spirit of their discussion, person-to-person assumption is appropriate when worker i can sufficiently summarize her situation and action taken, and worker j knows what appropriate actions should be for all his tasks so long as he understood the nutshell of worker i's situation and action. A common example is worker i informs j when to expect his work to be done, as a pilot would do to inform his crew and ground people about the plane's arrival time or an assembly worker informs others of a delay. In Vayanos (2003), a financial analyst provides summarized information to his supervisor, with some loss involved. Person-to-person communication is assumed by Ichniowski and Shaw (2004) and in Garicano (2000) who

$$Q = \sum_i \sum_{j \neq i} (a_{ij} - a_{jj})^2$$

Assume further that, at probability $r \in (0, 1)$, two workers can understand each other correctly and coordinate properly between them. In such an event, $(a_{ij} - a_{jj})^2 = 0$. At probability $(1 - r)$, they fail to understand each other and coordinate properly between them. This results in $(a_{ij} - a_{jj})^2 = \Delta a_{ij}$, $0 < \Delta a_{ij} < \infty$. We assume that this probability distribution is not affected by whether the communication is through a supervisor or directly between the workers, i.e., miscommunication is equally likely if it is made through a supervisor as it is directly between two workers.[8] Under this assumption, Q has the expected value of

$$\begin{aligned} E(Q) &= \sum_i \sum_{j \neq i} (1-r)\Delta a_{ij} \qquad (3) \\ &= n(n-1)(1-r)\Delta a_{ij} \\ &= [(1-b)/b^2](1-r)\Delta a_{ij} \end{aligned}$$

The second equal sign is based on the assumption of symmetry across jobs.[9]

It is easy to verify that $[\partial E(Q)/\partial b] < 0$, meaning that a larger job span leads to less aggregate coordination loss across workers. The idea behind this negative relationship between E(Q) and b is quite intuitive. Since a quality problem is due to misunderstanding between two workers, by reducing the number of workers involved in the production line, the firm can reduce the expected numbers of misunderstandings and quality problems.

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modeled workers looking for help from, respectively, fellow workers or supervisors to deal with randomly arriving problems.

[8] Involving a supervisor would lead us into a discussion of vertical specialization and that of supervisor's span of control. See Garicano (2000) for such a discussion.

[9] Note that Q can have two extreme values. One is $Q = Q_m = 0$, suggesting the highest quality due to no miscommunication at all between any two workers. This occurs at probability $r^{n(n-1)}$ if $n > 1$, and is guaranteed if $n = 1$. The other extreme value of Q is obtained when $n = N$, and all N workers misunderstand each other in every pair-wise communication. This happens at probability $(1 - r)^{N(N-1)}$. In such a case, $Q = Q_M = [(N - 1)N]\Delta a_{ij}$.

Since in the rest of this paper we will only be dealing with E(Q), for notational economy and without any risk of confusion, from here on, we use Q to denote E(Q).

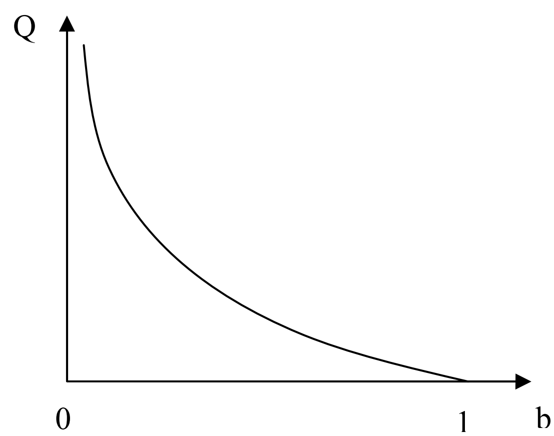
Define $\Delta = (1\text{-}r)\Delta a_{ij}$. The expected aggregate ineffectiveness measure Q can be rewritten as

$$Q = \Delta(1-b)/b^2 \tag{4}$$

We can reinterpret Q as a measure of quality of the product, with a larger (smaller) Q indicating poorer (better) quality.[10] Such a reinterpretation is natural. When the parts of a product have greater aggregate mismatch among them (seen in a larger Q), the total is less effective in serving the purpose. Under this interpretation, quality is the best at $Q=Q_m=0$. It declines as Q increases, and is worst at $Q=Q_M$.

From $Q = \Delta(1-b)/b^2$, we can derive $\partial Q/\partial b = \Delta(b-2)/b^3 < 0$ and $\partial^2 Q/\partial b^2 = \Delta(6 - 2b)/b^4 > 0$. The negative relationship between Q and b is illustrated in Figure 1.

Figure 1: The relationship between Q and b



Product demand

We assume a linear demand function with

[10] Dessein and Satos (1993) reinterpreted Q after the cost of communication as the firm's profit, with a smaller Q indicating a larger profit.

$$p = \psi(Q, I) - \omega q$$

where p is price, $I > 0$ consumer income, ψ a function of Q and I, $\omega > 0$ a constant, and q the quantity of the product sold.

Since a larger Q represents poorer quality, consumer demand for the product has a negative relationship with Q. Specifically, assume

$$\psi = \psi_m \geq 0, \quad \text{When } Q = Q_M$$

$$\psi = \psi_M > \psi_m \quad \text{When } Q = Q_m = 0$$

$\partial\psi/\partial Q$ exists for all $Q \in (0, Q_M)$, with $(\partial\psi/\partial Q) < 0$, and $(\partial^2\psi/\partial Q^2) < 0$, for all $Q \in (Q_m, Q_M)$; $(\partial\psi/\partial Q) = 0$ when $Q = Q_m$, and $(\partial\psi/\partial Q) = -\infty$ when $Q = Q_M$. These assumptions say that, for any given income, demand is at its lowest when quality is at its worst possible. The impact of better quality on demand is always positive. The impact is very strong when quality is extremely low, but declines and is zero when quality is at the highest level possible. Together, these assumptions guarantee an interior equilibrium level of quality, hence an interior solution for job design which determines quality.

We also assume that $\partial\psi/\partial I > 0$, and $\partial^2\psi/\partial Q\partial I \leq 0$, for all $Q \in (0, Q_M)$. $\partial\psi/\partial I > 0$ means that demand is larger at any price when income is higher. $\partial^2\psi/\partial Q\partial I < 0$ means that consumers become more quality sensitive when their income increases. In other words, at a higher income level, demand increases faster as quality improves. $\partial^2\psi/\partial Q\partial I = 0$ means that consumer sensitivity to quality remains the same at different income levels. The two situations are illustrated in, respectively, Figure 2a and Figure 2b below. In the figures, I_1 and I_0 are two income levels with $I_1 > I_0$.

Figure 2a: Case of $\partial^2\psi/\partial Q\partial I<0$

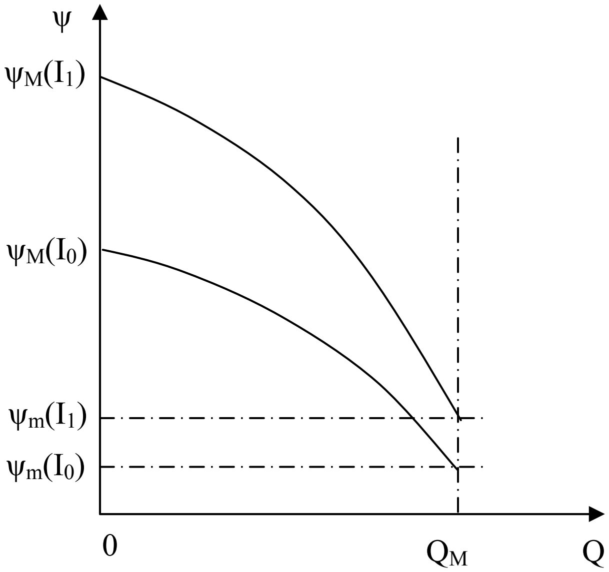
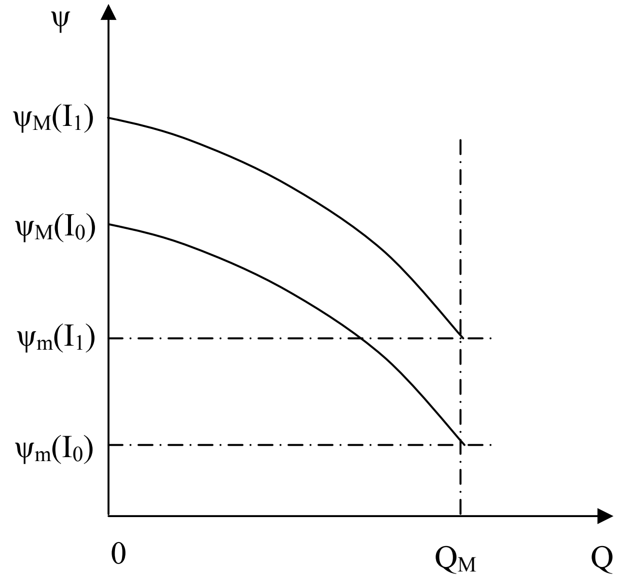


Figure 2b: Case of $\partial^2\psi/\partial Q\partial I=0$



Firm's objective

The firm's objective is to maximize profit, given by the difference between total revenue and total cost. Its total revenue is pq. Its total wage cost is wL, where w is wage and L total employment. Its total training cost is c(L/n) = cbL, where c is per line training cost and L/n the number of lines. Substituting these, the demand and the production functions into the firm's profit function, the firm's profit can be written as a function of two endogenous variables q and b as below.

$$\begin{aligned} \underset{q,\,b}{\text{Maximize}}\; E(\pi) &= E(pq) - wL - c(L/n) \qquad (5) \\ &= [E\psi(Q, I)]q - \omega q^2 - w\theta q^2 - cb\theta q^2 \end{aligned}$$

s.t. 1. All the constraints on the parameter values.

2. The technologies determining quality measure Q, output level q, and job span b.

In this optimization problem, the decisions variables are job span b (or the number of workers per production line), and total output level q (or the total number of workers hired L). The parameters are

I = income, which affects demand.

ω = slope of demand curve.

c = unit cost of training.

θ = a measure of productivity

r = probability of reliable and effective communication.

Δa_{ij} = significance of uncoordinated actions due to miscommunication affecting Q.

The main tradeoff in the model is between training cost and effectiveness of coordination. A larger job span reduces aggregate communication error (in probability terms), which improves expected product quality and thereby increases the demand for the product. But it necessitates more training for every worker, leading to a higher total cost of training.

3. Equilibrium.

The firm's maximization problem has first-order conditions with respects to b and q as below.

$$\partial E(\pi)/\partial b = (\partial E\psi/\partial Q)(\partial Q/\partial b) - c\theta q = 0 \qquad \text{FOC-b}$$

$$\partial E(\pi)/\partial q = E[\psi(Q, I)] - 2\omega q - 2w\theta q - 2cb\theta q = 0 \qquad \text{FOC-q}$$

The assumptions about $\partial E\psi/\partial Q$ guarantee that, when b is very small, Q is very large, and so is $(\partial E\psi/\partial Q)$. Thus the first term in FOC-b dominates, so that the whole expression is positive. When b is very large, the first term becomes very small so that the second term dominates and the whole expression is negative. Hence there exists a value of b at which FOC-b is satisfied. It is also obvious there exits a value of q at which FOC-q is satisfied. As shown below, the second-order conditions are also satisfied, suggesting that the optimal b and q both exist and are unique.

$$\partial^2 E(\pi)/\partial b^2 = \underset{(-)}{(\partial^2 E\psi/\partial Q^2)}\underset{(+)}{(\partial Q/\partial b)^2} + \underset{(-)}{(\partial E\psi/\partial Q)}\underset{(+)}{(\partial^2 Q/\partial b^2)} < 0$$

$$\partial^2 E(\pi)/\partial q^2 = -2(\omega + w\theta + cb\theta) < 0$$

Rearranging FOC-q yields

$$q^* = E\psi(Q, I)/2(\omega + w\theta + cb\theta)$$

Substitute q^* into FOC-b and rearrange the equation. The profit maximizing job span b^* is found when it satisfies the condition

$$(\partial E\psi/\partial Q)(\partial Q/\partial b) - c\theta[E\psi(Q, I)]/2(\omega + w\theta + cb\theta) = 0, \text{ or}$$

$$2(\partial E\psi/\partial Q)(\partial Q/\partial b)(\omega + w\theta + cb\theta) - c\theta[E\psi(Q, I)] = 0 ,$$

in which b^* is given as an implicit function of all the parameters.

For notational economy, from here on we will use ψ_Q in place of $\partial E\psi/\partial Q$, Q_b in place of $\partial Q/\partial b$, and ψ in place of $E\psi(Q, I)$. The condition for the equilibrium b is rewritten as

$$2\psi_Q Q_b(\omega + w\theta + cb\theta) - c\theta\psi = 0 \qquad (b^*)$$

The first term in b^* is the marginal benefit of a larger span of job seen in a smaller quality problem (Q_b) and, following it, an increased demand for the product (ψ_Q). The second term is the marginal cost of a larger job span.

4. Product demand and job span

In the model, the demand curve is determined by two parameters, the slope of the curve measured by ω, and income level I.

We first study how the slope of demand, measured by ω, affects b, i.e., we want to know the sign of $\partial b/\partial \omega$.

From (b^*), we obtain

$$2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta)(\partial b/\partial \omega) + 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta)(\partial b/\partial \omega)$$

$$+ 2\psi_Q Q_b + 2\psi_Q Q_b c\theta(\partial b/\partial \omega)$$

$$- c\theta\psi_Q Q_b(\partial b/\partial \omega)$$

$$= 0 .$$

Or, $A(\partial b/\partial \omega) = - 2\psi_Q Q_b < 0$, where

$$A = 2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta) + 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta) + c\theta\psi_Q Q_b$$

So it is clear that $\partial b/\partial \omega$ and its coefficient A have the opposite signs.

Proposition 1: Job span and the slope of demand have a positive relationship, i.e., $(\partial b/\partial \omega)>0$

Proof: Note that, in A, only the third term is positive. We prove that the third term is dominated by the second term so that A<0. To see this, Substituting $Q_{bb} = \Delta(6 - 2b)/b^4$ and $Q_b = \Delta(b-2)/b^3$ into the second and third terms in A, respectively, and define A' in these two terms.

$$A' = 2\psi_Q(\omega + w\theta + cb\theta)\Delta(6 - 2b)/b^4 + c\theta\psi_Q\Delta(b-2)/b^3$$

Drop ψ_Q in A', and multiply A' by b^4 to obtain

$$A'' = A'b^4/\psi_Q = 2(\omega + w\theta + cb\theta)\Delta(6 - 2b) + c\theta\Delta(b-2)b.$$

$A' < 0$ if $A'' > 0$ because $\psi_Q < 0$. Recall the fact that $0 < b \leq 1$. Replace b in the first term of A” with 1, and b in the second term of A” with 0 to obtain the inequality

$$A'' > 2(\omega + w\theta + cb\theta)\Delta 4 - 2c\theta\Delta b$$

$$= 8\Delta(\omega + w\theta) + 8\Delta cb\theta - 2\Delta cb\theta$$

$$= 8\Delta(\omega + w\theta) + 6\Delta cb\theta > 0. \qquad \textbf{Q.E.D.}$$

The result of Proposition 1 is not very intuitive. The reason for this result lies in the way in which the demand curve is specified. Under our specification of a linear demand, the demand curve declines faster with a larger ω. This means that, given ψ, demand is smaller at any price. From the equation for q^*, we can see that a larger ω indeed leads to a smaller optimal output, holding job design constant. From FOC-q, we can see that a smaller q means that the marginal total cost of training is smaller. This gives the reason for increased job span and more training associated with it when ω is larger. Hence we find a positive relationship between ω and b.

Next, we study how changes in income affect job design. When income increases, demand for the product goes up at any price and quality level. Should the firm increase b to better assure quality or do the opposite? The answer to this question depends on if the size effect or quality sensitivity effect of income dominates, as given by Proposition 2. By size effect, we mean how much demand increases with income, at any price and product quality. By quality sensitivity effect, we mean how much more the consumer is willing to pay for better quality when income is higher.

Proposition 2:

1) When market size effect of income dominates, a higher income leads to smaller job span, i.e., $\partial b/\partial I < 0$.

2) When quality sensitivity effect of income dominates, higher income leads to a larger job span, i.e. $\partial b/\partial I > 0$.

Proof: Again, from b^*, we can obtain

$$2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta)(\partial b/\partial I) + 2\psi_{QI}Q_b(\omega + w\theta + cb\theta)$$

$$+ 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta)(\partial b/\partial I) + 2\psi_Q Q_b c\theta(\partial b/\partial I) - c\theta\psi_I$$

$$= 0$$

which after rearranging is $A(\partial b/\partial I) = B$, where

$$A = 2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta) + 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta) + 2\psi_Q Q_b c\theta$$

$$B = c\theta\psi_I - 2\psi_{QI}Q_b(\omega + w\theta + cb\theta),$$

$A < 0$ as explained in the proof of Proposition 1. This means that the sign of $\partial b/\partial I$ is the opposite of the sign of B. In B, there are two terms with opposite signs. ψ_I in the first term measures the effect of income on demand, i.e., how much demand is increased when income is higher. Call it the market size effect of income. ψ_{QI} in the second term measures the effect of income on consumer sensitivity to product quality, i.e., how much consumer's preference for quality changes as income increases. Call this quality sensitivity effect of income. It is clear from B that, when the market size effect dominates, $B > 0$ and $(\partial b/\partial I) < 0$. When quality sensitivity effect dominates, $B < 0$ and $(\partial b/\partial I) > 0$. **Q.E.D.**

The intuition of Proposition 2 is not hard to understand. Adam Smith (1937) believed that the division of labor is limited by the extent of the market. Proposition 2.1 confirms this result. The mechanism leading to this result is explained by the equation for q^* and FOC-b. In a larger market, equilibrium output level is higher. In FOC-b, q is a multiplier to unit training cost c. This means that marginal total training cost increases faster when output level is higher. Holding the marginal gain from a larger job span and quality improvement constant, a larger output in a larger market leads to a reduced job span. It is not surprising at all that the result is reversed if at a higher income level consumers are willing to pay much more for better quality.

5. Labor costs and job span.

In this model, there are two types of labor costs: wage and training cost. We show that a higher wage unambiguously leads to a larger job span. Surprisingly, the effect of a higher unit training cost c on job design is not as clear-cut as one might think.

Proposition 3: Job span is larger when wage is higher, i.e., $\partial b/\partial w > 0$.

Proof: From b*, we obtain

$$A(\partial b/\partial w) = - 2\theta\psi_Q Q_b < 0.$$

where $A = 2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta) + 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta) + c\theta\psi_Q Q_b$. Since $A < 0$, as explained in the proof of Proposition 1, we have $\partial b/\partial w > 0$. **Q.E.D.**

Two facts are responsible for this result. First, in FOC-b, we can see that w has no direct effect on the optimal job span b. Second, in the equation for q^*, we can see wage adversely affects output level q, i.e., a larger w leads to a smaller q^*. So w affects b only through its effect on q. When output is reduced due to a higher wage, the marginal total cost of training is lower. It is thus worthwhile to increase job span and, through it, improve product quality and increase the demand for the product.

To see the empirical implication of Proposition 3, imagine a hypothetic situation of two plants identical in every respect, e.g. the technology, the amount of capital they use and the market they serve, etc., except that one of them is on the Mexican side of the border and the other on the American side. Proposition 3 predicts that, compared to their American counterparts, the workers in the Mexican plant will receive less extensive training, perform jobs of narrower task spans, and produce the same product at a higher output, but lower quality level, because wage is lower in Mexico. Moving beyond this hypothetic situation, it seems a fairly common phenomenon in many markets that products from developing countries would take the lower end of the quality spectrum, whereas the same product produced in developed countries would take the higher end. It is not hard to think of multiple explanations for this phenomenon. One of them is that domestic consumers in developed countries have a stronger preference for quality. This possibility is reflected in Proposition 2.2 above. Proposition 3 provides another reason, which has the empirical implication that expensive labor should not be used to produce low quality products. Instead, they should be more extensively trained to produce high quality product.[11]

Proposition 4: Job span is reduced when unit training cost is higher, i.e., $\partial b/\partial c < 0$, so long as unit training cost is sufficiently small relative to the size of the market measured by ψ. The opposite is true if unit training cost is large relative to ψ.

Proof: From b^*, we can obtain $A(\partial b/\partial c) = \theta(\psi - 2b\psi_Q Q_b)$, where

$$A = 2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta) + 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta) + c\theta\psi_Q Q_b < 0,$$

[11] Still another possible explanation is better education and general skill level of workers in developed countries. This explanation is a plausible for a given and limited period of time. However, in the longer run, one should probably think of the provision of education and acquisition of general skills as a response to the need for them. Proposition 3 predicts that the demand for training is greater when wage is higher and workers are used to perform enriched jobs and produce high quality product. Still another possible explanation is better management in developed countries. This argument to a large extent boils down to better coordination and is reflected in Proposition 5 below.

as shown in the proof of Proposition 1. So the sign of $\partial b/\partial c$ is the opposite of the right-hand side of the equation. In the right-hand side, there are two terms. $\psi > 0$ is from the demand function and measures the size of the market. $\psi_Q Q_b > 0$ in the second term measures the marginal effect of increased job span on demand. When c is not too large, $\psi_Q Q_b$ is not too large either. Given this, the first term will dominate if the size of the market measured by ψ is sufficiently large. The right-hand side of the equation is positive, giving the results in the proposition. **Q.E.D.**

Several things can be said about this proposition. First, intuitively, one would anticipate a higher unit training cost to unambiguously and straightforwardly lead to a narrower job span. But the proposition says not so. To understand the cause for the ambiguity, note that training cost has two effects on b^*. The first and direct effect is reflected in FOC-b where the optimal b is determined holding output q as constant. This effect says that b should always decrease when c is higher. However, like wage cost w, high unit training cost c also adversely affects the optimal output level q, as clear from the equation for q^*. When q from FOC-q is substituted into FOC-b, the indirect effect of c on b through its effect on q is reflected in the solution for b^*. As in the case of higher wage, when higher unit training cost c leads to a lower output, the indirect effect of it is to increase job span. Notice further that the effect of c on q is not constant. It is easy to verify by taking the first and second derivatives of q^* with respect to c that $q'(c)<0$, $q'(c)\rightarrow 0$ as $c\rightarrow 0$, and $q''(c)<0$, i.e., the effect of c on output is trivial when c is very small, but accelerates when c becomes larger and larger. Thus the indirect effect of c on b (through q) is limited when c is small and the direct effect of it dominates. So, when c is small, b is large (so that $\psi_Q Q_b$ is also small) and a negative relationship between c and b is found. But, when c becomes larger and larger, its negative effect on q accelerates and indirect effect on b also becomes stronger. Then the indirect effect might dominate, resulting in a positive relationship of c with b.

Technically, we can note that the combination of a small unit training cost c and a large market (relative to c) measured by ψ is not only reasonable, but also easily satisfied with certain parameter values, since c can always be arbitrarily close to zero. To the contrary, a large c (leading to a small b and thus large $\psi_Q Q_b$) relative to ψ could violate the non-negative profit condition for the firm. So the combination of a large c and a small ψ can be an empty set. However, we cannot be definitive about this without further specification of ψ.

6. Coordination technology, labor productivity and job span.

The model has three parameters measuring technological conditions. Among them, r is a measure of communication technology i.e., how reliable communication is between two workers. Δa_{ij} is a measure of damage due to miscommunication and mismatched actions between workers i and j. θ is a measure of productivity of the intermediate product $q_{\text{all lines}}$ produced by L. In this section, we investigate how these technological factors affect job design.

We have defined $\Delta = (1\text{-}r)\Delta a_{ij}$. Δ can be seen as a measure of how effectively two workers can coordinate between their actions. A larger Δ suggests less effective coordination. A line becomes less effective when communication is less reliable (i.e., when r is smaller), or miscommunication has a more severe consequence (i.e., when Δa_{ij}

is larger). Since any effect of r or Δa_{ij} has on job design is through its effect on Δ, here we focus on the effect of Δ on job design, i.e., we want to know the sign of $\partial b/\partial\Delta$.

Proposition 5: Worse coordination technology leads to larger job span, i.e., $\partial b/\partial\Delta > 0$.

Proof: Recall the fact that Δ is only in Q in the form of $Q = \Delta(1-b)/b^2$. In b^*, take derivative of b with respect to Δ to obtain

$$2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta)(\partial b/\partial\Delta) + 2\psi_{QQ}Q_\Delta Q_b(\omega + w\theta + cb\theta)$$
$$+ 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta)(\partial b/\partial\Delta) + 2\psi_Q Q_{b\Delta}(\omega + w\theta + cb\theta) + 2\psi_Q Q_b c\theta(\partial b/\partial\Delta)$$
$$- c\theta\psi_Q Q_\Delta - c\theta\psi_Q Q_b(\partial b/\partial\Delta)$$
$$= 0$$

Rearrange this expression to obtain $A(\partial b/\partial\Delta) = B$, where

$$A = 2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta) + 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta) + \psi_Q Q_b c\theta \text{ , and}$$

$$B = c\theta\psi_Q Q_\Delta - 2\psi_{QQ}Q_\Delta Q_b(\omega + w\theta + cb\theta) - 2\psi_Q Q_{b\Delta}(\omega + w\theta + cb\theta).$$

We know $A < 0$, as shown in the proof of Proposition 1. It is easy to verify that $Q_\Delta > 0$ and $Q_{b\Delta} < 0$. So $B < 0$, and $\partial b/\partial\Delta > 0$. **Q.E.D.**

Proposition 5 confirms our intuition that, when it is more difficult to coordinate between workers' actions, jobs should be less specialized to relieve the burden of coordination. Recall the fact that a larger Δ can be the result of either a smaller r, or a larger Δa_{ij}, or both. So, what Proposition 5 tells us is that, when communication becomes more difficult and less reliable (a smaller r), job span should increase. Similarly, if a misunderstanding and uncoordinated actions have a more severe consequence, job span should increase to improve coordination. The result confirms what Bolton and Dewatripont (1994) and Garicano (2000) have found before. But it is different than Dessein and Santos (2003) who modeled the tradeoff between *ex ante* and *ex post* coordination and found specialization to be non-monotonic with improvement in communication technology.

Next we study how a change in θ, which is a measure of labor productivity, affects job design. Recall the fact that the production of the final product has an inverse functional form of $L = \theta q^2$, with $\theta = 1/\theta' = 1/aK^{\alpha}$. This means that a larger θ represents lower productivity of the intermediate product $q_{\text{all lines}}$ and the labor used to produce it.

Proposition 6 : When labor productivity is lower, job span is larger, i.e., $\partial b/\partial\theta > 0$, if the output effect of productivity dominates. The opposite is true, i.e., $\partial b/\partial\theta < 0$, if the total training cost effect of productivity dominates.

Proof : In b^*, take derivative of b^* with respect to θ to obtain $A(\partial b/\partial\theta) = B$, where

$$A = 2\psi_{QQ}Q_b^2(\omega + w\theta + cb\theta) + 2\psi_Q Q_{bb}(\omega + w\theta + cb\theta) + \psi_Q Q_b c\theta, \text{ and}$$

$$B = -\,2\psi_Q Q_b(w + cb) + c\psi$$

$A<0$ is shown in the proof of Proposition 1. In B, the first term measure the effect of a change in θ on output. When this effect dominates, $B < 0$, leading to $\partial b/\partial\theta > 0$. The second term in B is positive. It measures how training cost increases when productivity declines. When this effect dominates, $\partial b/\partial\theta < 0$. **Q.E.D.**

So, workers are given broader jobs if output level is significantly lower because labor is less productive. However, if output level remains high (is not reduced by much) when labor is less productive, many more workers need to be hired. The marginal effect of a saving on unit training cost on total training cost becomes much more significant. This leads to reduced job span.

7. Discussions of the model

Marginal productivity and the effects of demand and supply on job design

We have explained in section 2 why it is necessary and reasonable to assume constant marginal productivity of labor on each production line. We argued that equilibrium job span can never be in the range where marginal productivity of labor is rapidly increasing or decreasing. When it is, the problem of job design would be trivial: the firm would simply add more workers to the line if the gain from assigning more workers to the line is large, or reduce the number of workers if the opposite is true.

When $q_{\text{all lines}}$ is used as an intermediate input to produce the final output, we assumed a Cobb-Douglas production function in which $q_{\text{all lines}}$ has declining marginal productivity. This assumption is conventional. It is also critical in the model and for many of its results. If the marginal productivity of $q_{\text{all lines}}$ (and L used to produce it) is constant, output level, and all the demand and supply factors affecting it, would have no effect on job span. We would then be in the special case where the division of labor is determined mainly by the need of coordination as in Becker and Murphy (1992).

To see the critical importance of declining marginal return of $q_{\text{all lines}}$ to the model, suppose that the production function is $q = aK^{\alpha}(q_{\text{all lines}})^{\beta}$ with $\beta = 1$ so that the marginal productivity of $q_{\text{all lines}}$ is constant. Then, since labor used in the production of $q_{\text{all lines}}$ also has constant marginal return, final output q would have a linear relationship with labor. Equation 2 becomes $L = \theta q$. The firm's objective function Equation 5 becomes

$$[E\psi(Q, I)]q - \omega q^2 - w\theta q - cb\theta q.$$

The first-order condition for optimum requires finding $b = b^*$ at which

$$\psi_Q Q_b q - cb\theta q = 0,$$

which after canceling terms is

$$\psi_Q Q_b - cb\theta = 0.$$

Job span is then purely determined by the tradeoff between product quality and the cost of training. We thus have

Proposition 7 : Output level does not affect job span if the marginal productivity of labor is constant.

Proof : See the preceding discussion. **Q.E.D.**

Intuitively, Proposition 7 is due to the fact that, when marginal productivity of labor is constant, the marginal benefit of increased job span and the marginal cost of it are weighted equally by q. As the weights of q on benefit and cost of increased job span are exactly the same and cancels out with each other, q becomes neutral in determining job design. With output level irrelevant, factors such as consumer income, slop of demand and worker's wage, which all affect job span only indirectly through output level, would no longer have any effect on job design. Unit training cost and labor productivity still affect job design, but the ambiguity of their effects are removed because the indirect effects on job span through changes in output level are no longer there.

Since the effect of output on job span is neutral only when it has exactly equal weight on both the marginal benefit and the marginal cost, the neutrality goes away when the productivity of $q_{\text{all lines}}$ is not constant. For this reason, we know the assumption of β=0.5 is not critical. So long as $0 < \beta < 1$, all the results of our model are qualitatively unchanged. When $\beta > 1$, meaning that labor has an increasing return, output has a larger weight on the marginal benefit of job design than on marginal cost. The results of our model would then be reversed. A larger market and lower wage would have led to an increased job span, for example, because a higher level of equilibrium output makes the marginal total benefit of increased job span more significant.

In their model, Becker and Murphy (1992) assumed that every worker is equally productive, which implies constant marginal productivity of labor. They then focused on the optimization problem of the individual worker, with individual worker's productivity and the problem of coordination as the main tradeoff involved in the firm's specialization decision. As clear from Proposition 7, this assumption is critical for their conclusion that the size of the market generally has no effect on specialization.

It is worthwhile and important to point out that, to say that output affects job span negatively or positively does not mean that job span will necessarily decrease or increase with output. As clear from Propositions 2, 4 and 6, sometimes, a countervailing force may exist and completely offset the effect of a larger output on job span.

Integer problem.

Since the number of workers n is an integer and that the production line is of unit length, we face the problem that, after the optimal job span b^* is determined, it is likely that $1/b^* \neq n$, or, equivalently, $nb^* \neq 1$. This problem does not fundamentally affect our analyses. It can be dealt with in different ways.

Let n be the number of the workers assigned to a line such that $(n-1)b^* < 1$, but $nb^* \geq 1$. We can let b^* be the job span of (n-1) workers so that they can cover $(n-1)b^*$ of the line. The nth worker can then be assigned to the last job to fully cover the production line. The job span of this worker is $b_n = [1 - (n-1)b^*] \leq b^*$.

We can arbitrarily assume that the last worker, who serves as a residual taker, cannot affect product quality so that all our analyses could proceed as before. If the coordination of the nth worker with everybody else is equally important, then a marginal change in job span b may or may not affect product quality measure Q, for Q can be improved only when the change in job span b is sufficiently large so that one worker is cut from the line. Let $b_0 < b_1$. A change in job span from b_0 to b_1 leads to better product quality only if $(n-1)b_0 < 1$ (so that under b_0 the nth worker is needed to take up the residual job) and $(n-1)b_1 \geq 1$ (so that under b_1 the nth worker is not needed). As the effect of job span on Q becomes discrete, so does the tradeoff between quality and the cost of training.

Discrete changes in Q should not fundamentally affect the analyses and the results of our model. In the model, the effects of market size, worker's wage, the slope of demand and coordination technology on job span are all monotonic. So, instead of continuous adjustment of job span (in the absence of any adjustment cost), the firm will wait until a parameter, e.g., market size, has sufficiently changed to warrant a significant change in job span that will lead to a change in the number of workers needed to fully cover the line. The direction of such a discrete change in job span would be the same as in the case of continuous changes that we have analyzed.

The same is true when unit training cost c is sufficiently small relative to the size of the market. When c is large and changes to cross a turning point, the direction of its effect on job span may be difficult to predict. The significance of this problem, however, is likely to be limited for two reasons. First, the number of such turning points is limited. Second and more importantly, since in most realistic situations, unit training cost c is very small relative to the size of the market, we can restrict our attention to the range of the values of c in which $\partial b/\partial c < 0$. Within the range, discrete change in job span would have the same direction predicted by the analysis of continuous change in b. Labor productivity measure θ is another parameter with an ambiguous effect on job span so that a change of it crossing the turning point may make it difficult to predict the direction of the change in job span. Again, we can notice the limited number of such turning points and can use the result of Proposition 6 to predict discrete changes in job span within a range of productivity measure θ.

Other benefits of increased job span

Empirical studies of human resource practices have consistently found that job design affects performance in multiple dimensions, such as delivery time and product varieties, besides product quality. Our model can be reinterpreted to accommodate other performance measures.

We can see delivery time as a quality issue, and reinterpret Q as a measure of delay time in delivering the product. Informing everybody else if a bottleneck is being formed and what kind of help is needed to resolve the problem, and when to expect the work to be done, are often important contents of communication and coordination among the workers working together. It is also a typical case of person-to-person communication. Failure to effectively transmit this kind of information and take proper actions causes a delay in product delivery and is costly to the firm like other quality problems. In the words of our model, (1-r) can be the probability that two workers misunderstand each other. Δa_{ij} can be the extra time lost between two workers when they experience a coordination failure. Q can be the aggregate time loss. Empirically, the studies of Baily (1993), Berg et al (1994) and Dunlop and Weil (1996) all provided convincing evidence that mitigating such time losses is why in the apparel industry group-based job design (in which each worker performs a much larger span of tasks) is more likely to be adopted when the pressure on delivery time is high.[12]

A very important feature of modern manufacturing is variety proliferation of many products. Firms strive for a strategy of increased varieties typically design jobs more broadly for their workers.[13] Although it does not directly address this phenomenon, our model sheds light on the relationship of product variety to job design. We can see increased product varieties as a firm moving away from serving one large market of mass consumption to serving more smaller markets. Our model has a clear prediction that this should lead to a larger job span. We can also see increased varieties as a quality issue, with customer satisfaction improved when the variety for what a customer exactly wants is more readily available. When customers demonstrate greater willingness to pay a premium for the varieties of their individual choices, we can say their quality sensitivity has increased. Our model again predicts increased job span as a response.

In another angle, it is very reasonable to see consumer sensitivity to variety as a normal good, similar to what we have assumed about consumer sensitivity to product quality.[14] Thus we can expect the demand for product variety to become stronger as consumer income increases. This has the implication that, when product variety is considered, it may no longer true that higher income leads to a larger market (more demand for *the* product at given prices). In fact, the opposite may be true. As higher income leads to increased customer willingness to pay for the variety of one's individual

[12] Berg et al (1994) specifically noted that, under the progressive bundle system featured by narrowly defined jobs, bundles moving from one operator to another are often not picked up on time, but rather waiting at each station.

[13] See Aoki (1990) for an account of the relationship between product variety and job design in the Japanese automobile industry, and Ichniowski et al (1997) for an account of this relationship in the US steel industry.

choice, it serves as a market divisor: At higher consumer incomes, the firm finds itself serving not one larger, but many smaller markets. In such a case, consumer income would no longer have opposing effects on job span as found in Proposition 2. Rather, it would unambiguously lead to increased job span.[15]

8. The relationship with the literature

This paper is most closely related to the literature studying coordination problems in organizations, especially that in the tradition of Becker and Murphy (1992), Bolton and Dewartripont (1994) and Dessein and Santos (2003) on horizontal division of labor. In the Introduction, we summarized Becker and Murphy (1992). The main tradeoff in Bolton and Dewartripont (1994) is that workers become more efficient when they handle only specialized information, which provides the reason for assigning more workers to parallel processing of a cohort of information. In our model, assigning more workers to a production line has a comparable effect on efficiency reflected in savings on training cost. In their model, the benefit of specialization is (partly or entirely) offset by increased costs of communication when more workers work together and form a communication network. In our model, the benefit is also offset by potential communication problems leading to lower expected product quality. Dessein and Santos (2003) modeled the tradeoff between *ex ante* coordination by rules and *ex post* coordination by communication, and endogenously derived the demand for coordination. As already pointed out, the part of our model on coordination is directly from theirs.

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[14] Hodaka Morita and Michael Waldman called my attention to this point.

[15] Whether higher consumer income will lead to a larger market or greater demand for varieties is an empirical question. Historically, both types of experiences seem to exist.

Other important contributions to the literature of coordination include Marschak and Radner (1972), Arrow (1974), Radner (1992, 1993), Van Zandt (1990, 1999), Radner and Van Zandt (1992), and Vayanos (2003), who studied information processing and decentralization. Hierarchy is often the subject of these studies, whereas this paper considers only horizontal division of labor. Ichniowski and Shaw (2004) studied how connective capital possessed by workers affects knowledge sharing and problem solving.

A large literature has studied various incentive problems and their implications for organizational design and human resource practices.[16] Our model abstracts away from all incentive issues. It would be very interesting to see what further insights can be gained in a more integrated, but still manageable, model that incorporates the firm's external demand and supply conditions, internal coordination and incentive problems.

Human capital theory is a cornerstone of modern labor economics. It also has many important implications for organizations. Theories of human capital investment can be intertwined with either incentive or pure coordination problems.[17] Among many others, Kahn and Huberman (1988), Chang and Wang (1996) and Bai and Wang (2003) studied moral hazard problems in human capital investment and their implications for organizational design. Waldman (1990) studied the signaling effect of human capital investment. Chang and Wang (1995) and Acemoglu and Pischke (1998) used models with both moral hazard and selection problems in human capital investment to derive multiple equilibria in human resource practices. Bernhardt and Scoones (1998) studied

[16] Ben-Ner at al (1993) have a discussion of coordination and incentives as the two fundamental problems in organizations. Maskin, Qian and Xu's (2000) work on multidivisional versus unitary hierarchies in providing information on managerial performance is a good example of incentive problems with organizational implications. A survey of the incentive literature would be a very challenging task. Predergast (1999) provides one that is concise and accessible.

how labor market competition under asymmetric information and internal promotion affect worker's specific capital investment decision. Ichniowski and Shaw (2004) is an example of human capital investment to address coordination problems. Our model has per worker and total amount of training as a function of job design. So its results on job design contribute to the human capital theory.

International differences in the organization of production and human resource practices have interested many scholars. Studies along this line not only are applications of, but also contribute greatly to, organizational theories. Drastically different US and Japanese human resource practices captured much interest in these studies. Aoki (1988) provides an authoritative survey of this literature. Recent contributions to this literature include Carmichael and MacLeod (1993), Chang and Wang (1995), Blinder and Kruger (1996), Kato (2001), Morita (2001, 2003, 2004). The comparative statics analyses of our model have implications for comparative studies of organizations.

Morita (2004) has noticed that, in most theoretic studies of organizations and human resource practices, firm's strategic behavior and employment practices have been treated separately under industrial-organization and labor-centered models. Recently, a new trend has emerged to more explicitly study the relationship of the external market with the internal organization and human resource practices. Prominent in this minority group is the pioneering work of Hermalin (1994) on how Cournot competition affects incentives for managers. Also in the literature are Wang (2002) on the effect of spatial competition on firms' multiskilling decision, and Morita (2004) on the effect of firm's

[17] Human Implications of human capital investment for organizations including comparative organizations have been extensively studied. See Gibbons and Waldman (1999) for a survey.

survival rate on labor mobility and specific human capital investment.[18] This paper is another effort in this direction that takes a more integrated approach to the study of the interdependence and interactions between market and internal organization.

9. Summary and concluding remarks

Our model is built on the following few plausible and conventional core assumptions.

- There is a gain to specialization.
- Coordination is more difficult when jobs are more specialized, leading to lower product quality.
- The demand for the product is lower when the price of the product is higher, the income of the consumer is lower, or the quality of the product is poorer.
- The marginal productivity of labor is decreasing.

An integrated treatment of these factors fills a gap between empirical literature and theories studying job design. It allows the model to shed much new light on previous theories of what determines the division of labor. The model confirms some results obtained by other authors, such as how coordination technology affects job design. It clarifies some issues, such as the effect of market size on the division of labor. It has also obtained new results on how consumer sensitivity to product quality, labor cost, and labor productivity affect job design.

The model has the capacity to be extended. The desirable directions of extension discussed below also reflect the limitations of the present model. First, it would be interesting and important to model a market in which there is more than just one firm.

[18] Also worth mentioning is Milgrom and Roberts (1990), whose incorporate a downward sloping demand curve into the model on manufacturing strategy.

Such a model can be used to study how strategic interaction among the firms affects job design. For example, if, besides price, quantity and capacity, the firms also compete in product quality for market shares, how is the tradeoff between the cost and benefit of specialization is affected? How do the firms in the same market differentiate themselves in their competitive (e.g., quality) strategy, job design and other dimensions of internal organization? These are questions of great theoretic and empirical importance.

Second, in reality, job design must typically consider both coordination and incentive problems. However, so far, we have to yet build a model that includes both fundamental problems in organizations. Technically, modeling either one of these two fundamental organizational problems is complicated enough. An integrated treatment of them both would make the model difficult to track. The model we used in this paper is meaningful and yet not too hard to track. Hopefully, it represents a step forward towards a model of job design that contains both coordination and incentive problems.

References:

Acemoglu, D., and Pischke, J., 1998. “Why Do Firms Train? Theory and Evidence”, *Quarterly Journal of Economics*, vol.113, pp.79-119.

Aoki, Masahiko, 1988. “Information, Incentives, and Bargaining in the Japanese Economy.” *Cambridge University Press*.

Aoki, Masahiko, 1990. "Toward an Economic Model of the Japanese Firm." *Journal of Economic Literature*, v. XXVIII (March), pp.1-27.

Arrow, K J., 1974. “The Limits of Organization.” *New York, NY Norton*.

Bai, Chongen and Wang, Yijiang, 2003, "Uncertainty in Labor Productivity and Specific Human Capital." *Journal of Labor Economics*, vol. 21(3), July, pp.651-675.

Bailey, Thomas 1993. “Organizational Innovation in the Apparel Industry.” *Industrial Relations 32 (Winter),* pp.30-48.

Becker, G., and Murphy K., 1992. “The Division of Labor, Coordination Costs, and Knowledge.” *Quarterly Journal of Economics*, vol CVII, pp.1137-1160

Bedeian, Arthur G. and Raymond F. Zammuto, 1991. *Organizations: Theory and Design*. The Dryden Press.

Ben-Ner, Avner, Michael Montias, and Egon Neuberger, 1993. "Basic Issues in Organizations: A Comparative Perspective." *Journal of Comp. Economics*, June, 17., pp.207-42.

Berg, Peter, Eileen Appelbaum, Thomas Bailey, and Arne Kalleberg, 1994. “The Performance Effects of Modular Production in the Apparel Industry.” *Economic Policy Institute Mimeo.*

Bernhardt, Dan, and David Scoones, 1998. “Promotion, Turnover, and Discretionary Human Capital Acquisition.” Journal of Labor Economics, January, vol. 16, no. 1, pp. 122-141.

Blinder, A. and Kruger, A., 1996. “Labor Turnover in the USA and Japan: A tale of Two Countries.”, *Pacific Economic Review*, vol 1, pp.27-57

Bolton, Patrick and Dewatripont, Mathias, 1994. “The Firm as a Communication Network.” *The Quarterly Journal of Economics*, vol CIX, Issue 4, pp.809-839

Carmicheal, H.L. and Macleod, W. B. 1993. “Multiskilling, Technical Change and the Japanese Firm.” *The Economic Journal*, 103:142-160

Chang, C., and Wang, Y., 1995. "A Framework for Understanding Differences in Labor Turnover and Human Capital Investment." *Journal of Economic Behavior and Organization*, vol. 28, pp.91-105

Chang, Chun, and Wang, Yijiang. 1996. "Human Capital Investment under Asymmetric Information: The Pigovian Conjecture Revisited." *Journal of Labor Economics* 14:505-519.

Crémer, J., 1980. "A Partial Theory of the Optimal Organization of Bureaucracy." *Bell Journal of Economics*, vol XI, pp.683-693

Dessein, W. and Santos, T. 2003. "The Demand for Coordination." *NBER Working Paper*, No. w10056.

Dunlop, John and David Weil, 1996. "Diffusion and Performance of Modular Production in the U.S. Apparel Industry." *Industrial Relations*. July, 35:3, pp. 334-54

Garicano, L., 2000. "Hierarchies and the Organization of Knowledge in Production." *Journal of Political Economy*, vol. 108, pp.874-904

Geanakoplos, J., and Milgrom, P., 1991. "A Theory of Hierarchies Based on Limited Managerial Attention." *Journal of the Japanese and International Economies*, vol V, pp.205-225

Geary, John F., 1999. "The New Workplace: Change at Work in Ireland." *International Journal of Human Resource Management*, 10:5 October:870-890.

Geber, Beverly, 1992. "Saturn's Grand Experiment." Training, June, pp. 27-35.

Gibbons, H. Robert, and Michael Waldman, 1999. "Careers in Organizations: Theory and Evidence." In *Handbook of Labor Economics*, Volume III, (O. Ashenfelter and D. Card, eds., pp.2373-2437. Amsterdam, The Neverlands: North-Holland.

Hermalin, Benjamin E., 1994. "Heterogeneity in Organizational Form: Why Otherwise Identical Firms Choose Different Incentives for their Managers." *Rand Journal of Economics*, vol. 25, no. 4, Winter, pp.518-537.

Ichniowski, Casey, and Kathryn Shaw, 2004. "Connective Capital: Building Problem Solving Networks within Firms." *Working paper*.

Ichniowski, Casey; Kathryn Shaw and G. Prennushi, 1997. "The Effects of Human Resource Management Practices on Productivity: A Study of Steel Finishing Lines." *American Economic Review* 87:291-313.

Kahn, Charles, and Huberman, Gur. 1988. "Two-Sided Uncertainty and 'Up-or-Out' Contracts." *Journal of Labor Economics* 6, no.4: 423-443.

Kato, T. 2001. "The End of Lifetime Employment in Japan? Evidence from National Surveys and Field Research." *Journal of the Japanese and International Economies*, vol.15, pp.489-514

Lazear, Edward P., 1998. "Personnel Economics for Managers." *New York : Wiley, c1998*

Marschak, J., and Radner, R., 1972. "Economic Theory of Teams." *New Haven, CT. Yale University Press*

Maskin, E., Qian, Y., and Xu, C. 2000. "Incentives, Information, and Organizational Form." *Review of Economic Studies*, 67: 359-378.

Milgrom, P. and Roberts, J. 1990. "The Economics of Modern Manufacturing Technology, Strategy, and Organization." *American Economic Review*, (June 1990): 511-528

Morita, Hodaka, 2001. "Choice of Technology and Labor Market Consequences: an Explanation of U.S.-Japanese Differences.", *Economic Journal*, 2001, vol. 111, pp.29-50

Morita, Hodaka., 2003. "Multiskilling, Delegation, and Continuous Process Improvement: A Comparative Analysis of U.S.-Japanese Work Organizations.", forthcoming in *Economica*

Morita, Hodaka, 2004. "Firm Dynamics, Labor Mobility and Specific Human Capital." *University of New South Wales, Working Paper*.

Osterman, Paul, 1994. "How Common is Workplace Transformation and Who Adopts it?" *Industrial and Labor Relations Review*, Vol.47, No. 2, 173-188.

Prendergast, Canice. "The Provision of Incentives in Firms." *Journal of Economic Literature*, 1999, 37(1), pp. 7-63.

Qian, Yingyi, 1994. "Incentives and Loss of Control in an Optimal Hierarchy." *Review of Economic Studies*, July, 61, pp.527-544.

Radner, R., 1992. "Hierarchy: The Economics of Managing." *Journal of Economic Literature*, vol XXX, pp.1382-1415

Radner, R., 1993. "The Organization of Decentralized Information Processing." *Econometrica*, vol LXI, pp.1109-1146

Radner, R., and Van Zandt, T., 1992. "Information Processing in Firms and Returns to Scale." in *Annales d'Economie et de Statistique*, vol. XXV-XXVI, pp.265-298

Smith Adam, 1776. "The Wealth of Nations."

Van Zandt, Timothy, 1990. "Efficient Parallel Addition." *Bell Laboratories Discussion Paper*.

Van Zandt, Timothy, 1999. "Real Time Decentralized Information Processing as a Model of Organizations with Boundedly Rational Agents." *Review of Economic Studies*, 66 (July), pp. 633-658.

Vayanos, Dimitri, 2003. "The Decentralization of Information Processing in the presence of Interactions." *Review of Economic Studies*, 77, pp.667-695

Waldman, Michael. 1990. "Up-or-Out Contracts: A Signaling Perspective." *Journal of Labor Economics* 8: 230-250.

Wang, Yijiang, 2002. "Product Market Conditions and Multiskilling." Mimeo, *University of Minnesota*.